

# Formal Verification of Next-Generation Airborne Collision Avoidance System with Adversarial Intruder Behavior

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## Background

- **Mid-air aircraft collisions** are increasing in likelihood as air space gets more congested.
  - Collision avoidance maneuvers must be performed as a last resort when two aircraft are on a collision course.
- It is imperative to **formally verify the safety of these maneuvers** in advance under all reasonable flight circumstances to ensure that no collisions happen.
- **ACAS X** is a collision-avoidance system developed by the Federal Aviation Association (FAA) [1].
  - Tracks the position and velocity of an ownship and intruders in its vicinity and uses this data in its collision avoidance advisories.
  - An advisory asks the pilot to maintain their vertical speed, or accelerate towards a new vertical speed.
  - Advisories only apply to the aircraft's climb rate in the vertical direction (not horizontal)

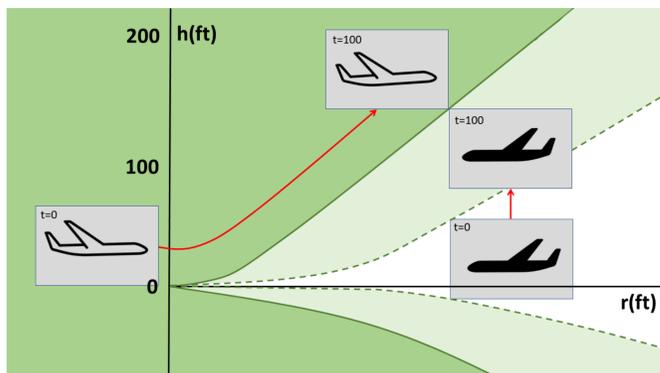


Figure 1: Safe region (dark green) and minimally compliant trajectory (red) with a vertically maneuvering intruder. The region that would be safe without intruder maneuverability is shown in light green.

## Introduction

- Previous work has assumed **severe restrictions on the intruder's maneuverability** in their formal verification of ACAS X, limiting their safety guarantees in real-world collision scenarios.
- Our previous work [2] used hybrid systems in its verification and introduced the idea of *safe regions*.
  - A region is safe if for all possible ownship positions and velocities within the region, a near mid-air collision (NMAC) will never occur (Fig. 1).
  - If ACAS X issues an advisory, and following this advisory always keeps the ownship within a safe region, then this advisory is guaranteed to be safe.
  - This work assumed that the intruder aircraft is approaching at a constant velocity, both horizontally and vertically.

## Contribution

- My work represents aircraft encounters with Differential Game Logic (dGL) [3] as opposed to the Differential Dynamic Logic [4] of our previous work.
  - dGL allows for the **modeling of an intruder aircraft that can change its flight path and velocity** as the encounter is evolving.
  - Hybrid games apply perfectly to collision-avoidance because the intruder may act adversarially, and the ownship must have a **strategy to overcome any sequence of intruder maneuvers**.
- This is the first work that we know of to apply hybrid games to the problem of collision-avoidance.

```

1 init() ∧ R(r, h, v, advisory = (w, vlo)) →
2 [( advisory := (w, vlo); ?R(r, h, v, advisory = (w, vlo)); )
3 (ao := ownship(advisory)); d
4 (ai := intruder ;
5 {motion(ao, ai) & EDC(v, vlo, a, alo)}
6 )*
7 ]*(¬NMAC)
    
```

Figure 2: high-level model showing the general structure of the models developed in this work

## Model Overview

- Formula  $P \rightarrow [\alpha]Q$  (Fig. 2) says that all executions of hybrid program  $\alpha$  starting in a state satisfying logical formula  $P$  end up in a state satisfying  $Q$ .

- Preconditions require ownship to be in a *safe region*  $R(r, h, v, advisory=(w, v_{lo}))$  for initial advisory  $(w, v_{lo})$ .

- Lines 2-4 encode the discrete choices of the advisory, followed by the ownship and intruder accelerations
  - Intruder choice (line 4) is omitted from models that do not allow for intruder maneuverability.

- Ownship choice (line 3) is contained in the  $(^d)$  operator, which differentiates choices made by the ownship and intruder.
  - All choices in the program  $(^d)$  are resolved by our helpful player, so we need only show that the pilot *can* strategically pick her acceleration such that for *any* advisory satisfying the safe region and *any* set of intruder actions, a collision does not occur.

- Motion equations (line 5) are followed for any duration of time, as long as the evolution domain constraint (EDC(v,a)) is true.

- Final \* operator means that the program can be repeated any number of times.
  - Postcondition guarantees that *any sequence* of advisories satisfying our safe region will be safe.

## Results

- **Developed and proved seven hybrid game models** verifying under appropriate assumptions that the ownship can always maneuver to safety.
- Each model was developed and verified using the theorem prover KeYmaera X.
- Model categories
  - **Infinite-horizon**
    - Requires advisories to be safe indefinitely
      - While the system can issue follow-up advisories, the aircraft must avoid collision *whether or not* a new advisory is issued.
    - This is a rigid definition of safety: in some scenarios it is better to follow an unsafe advisory for limited time, followed by an advisory that is safe indefinitely
  - **Finite-horizon**
    - Proves safety of advisories only up to  $\epsilon$ -time
    - No liveness: does not give formal safety guarantees after  $\epsilon$ -time
    - Stepping-stone to the safeable models
  - **Safeable**
    - An advisory is *safeable* if and only if it is safe or can still be made safe in the future, if necessary, via subsequent advisories.
    - Proves  $\epsilon$ -time safety with liveness: advisories are safe after  $\epsilon$ -time if there exists a follow-up advisory which is safe indefinitely
    - Example shown in Fig. 3

- Infinite-horizon models
  - Model 1: non-maneuvering intruder
  - Model 2: vertically-maneuvering intruder
  - Model 3: horizontally-maneuvering intruder

- Finite-horizon models
  - Model 4: non-maneuvering intruder
  - Model 5: vertically-maneuvering intruder

- Safeable models
  - Model 6: non-maneuvering intruder
  - Model 7: vertically-maneuvering intruder

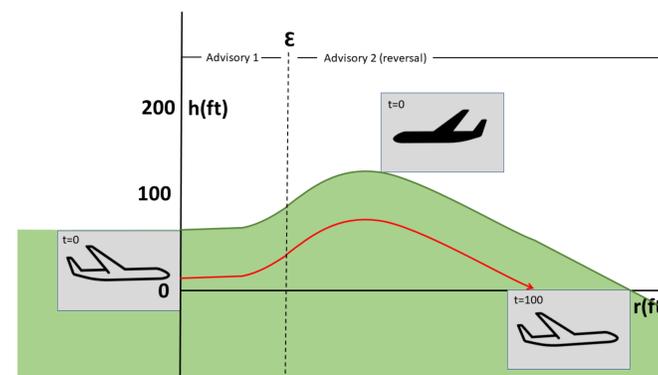


Figure 3: safe region (green) of an ownship following Advisory 1 for  $\epsilon$ -time and Advisory 2 afterwards

## Conclusion

- This work applied hybrid games to the verification of ACAS X
  - Proved that against an intruder with limited maneuverability, an ownship given a safe ACAS X advisory can always find a verifiably safe path.
- Challenges
  - Development of an ownship strategy
    - Ownship does not know intruder actions in advance, only the broad limitations of the intruder's maneuvering capabilities.
    - Strategy must ensure safety no matter what the intruder chooses to do.
  - Correct assignment of responsibility for each action
    - Positive outcomes of one actor act as negative outcomes for the other.
    - Vital to the correctness of the model that the discrete choices be resolved by the correct actor to prevent any unfair advantages.
    - Previous model iterations incorrectly allowed the choice of advisory to be resolved by the ownship, allowing the ownship to choose an *optimal* advisory instead of proving safety of *all possible* advisories.

## Future Work

- Develop and prove bounded-time and safeable models in which the intruder has horizontal maneuverability
- Develop models for an intruder which can simultaneously maneuver horizontally and vertically
- Ownship collision-avoidance maneuvers are currently limited to the vertical direction
  - Expand ownship maneuverability to the horizontal direction

## References

1. Federal Aviation Administration TCAS Program Office: Algorithm design description for the surveillance and tracking module of ACAS X (2014). Run12
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3. Platzer, A.: Differential game logic for hybrid games. Technical Report CMU-CS-12-105, School of Computer Science, Carnegie Mellon University, Pittsburgh, PA (2012).
4. Platzer, A.: Differential dynamic logic for hybrid systems. J. Autom. Reas. 41(2), 143–189 (2008).