

**1: Operators of First-order Intuitionistic Logic**

FOIL	KeYmaera I	Operator	Informal meaning
$p(e_1, \dots, e_k)$	<code>p(e1, ..., ek)</code>	predicate	arbitrary proposition may depend on $e_1, \dots, e_k$
$\top$	<code>true</code>	truth	is always true
$\perp$	<code>false</code>	falsehood	has no proof
$\neg A$	<code>!A</code>	negation / not	$A$ implies false
$A \wedge B$	<code>A &amp; B</code>	conjunction / and	both $A$ and $B$ are true
$A \vee B$	<code>A   B</code>	disjunction / or	evident that $A$ is true or evident that $B$ is true
$A \supset B$	<code>A -&gt; B</code>	implication / implies	truth of $A$ implies truth of $B$
$A \equiv B$	<code>A &lt;-&gt; B</code>	bi-implication / equivalent	$A$ implies $B$ as well as $B$ implies $A$
$\forall x A$	<code>\forall x A</code>	universal quantifier / for all	$A$ for all $x$ of type $\tau$
$\exists x A$	<code>\exists x A</code>	existential quantifier / exists	$A$ for some witness for $x$ of type $\tau$

**Functions.**

```

B p().          /* predicate symbol of no arguments */
B q().          /* predicate symbol of no arguments */
B r(R).         /* predicate symbol of one argument */
    
```

**End.**

**Problem.**

```

(p()|q()) -> (p()->\forall x r(x)) & (q()->\forall x r(x))
-> \forall x r(x)
    
```

**End.**

**Functions.**

```

B p().          /* predicate symbol of no arguments */
B q().          /* predicate symbol of no arguments */
B r().
B s().
    
```

**End.**

**Problem.**

```

p()&q() -> (q()->r()) -> (p()->(r()->s())) -> s()
    
```

**End.**

**Solution.**

```

/* tactic that proves this, here an explicit search-free tactic */
implyR(1) & implyR(1) & implyR(1) & andL(-1) & implyL(-2) & <(
  close ,
  implyL(-1) & <(
    close ,
    implyL(-2) & <(
      close ,
      close
    )
  )
)
)
)
    
```

**End.**

**2: Intuitionistic propositional sequent calculus proof rules**

$\text{andR } \wedge R \frac{\Gamma \rightarrow A \quad \Gamma \rightarrow B}{\Gamma \rightarrow A \wedge B}$	$\text{orR1 } \vee R_1 \frac{\Gamma \rightarrow A}{\Gamma \rightarrow A \vee B}$	$\text{notR } \neg R \frac{\Gamma, A \rightarrow \perp}{\Gamma \rightarrow \neg A}$
$\text{andL } \wedge L \frac{\Gamma, A, B \rightarrow C}{A \wedge B, \Gamma \rightarrow C}$	$\text{orR2 } \vee R_2 \frac{\Gamma \rightarrow B}{\Gamma \rightarrow A \vee B}$	$\text{notL } \neg L \frac{\neg A, \Gamma \rightarrow A \quad \perp, \Gamma \rightarrow C}{\neg A, \Gamma \rightarrow C}$
$\text{implyR } \supset R \frac{\Gamma, A \rightarrow B}{\Gamma \rightarrow A \supset B}$	$\text{orL } \vee L \frac{A, \Gamma \rightarrow C \quad B, \Gamma \rightarrow C}{A \vee B, \Gamma \rightarrow C}$	
$\text{implyL } \supset L \frac{A \supset B, \Gamma \rightarrow A \quad B, \Gamma \rightarrow C}{A \supset B, \Gamma \rightarrow C}$		
$\text{closeTrue } \top R \frac{}{\Gamma \rightarrow \top}$	$\text{closeFalse } \perp L \frac{}{\perp, \Gamma \rightarrow C}$	$\text{close init } \frac{}{P, \Gamma \rightarrow P}$

**3: Quantifier sequent calculus proof rules (uni-typed with existence presupposition)**

$\text{allR } \forall R \frac{\Gamma \rightarrow A(e)}{\Gamma \rightarrow \forall x A(x)} \quad (c \text{ new})$	$\text{existsR } \exists R \frac{\Gamma \rightarrow A(e)}{\Gamma \rightarrow \exists x A(x)}$
$\text{allL } \forall L \frac{\forall x A(x), \Gamma, A(e) \rightarrow C}{\forall x A(x), \Gamma \rightarrow C}$	$\text{existsL } \exists L \frac{A(c), \Gamma \rightarrow C}{\exists x A(x), \Gamma \rightarrow C} \quad (c \text{ new})$

**4: Admissible rules**

$\text{close id } \frac{}{A, \Gamma \rightarrow A}$	$\text{cut } \frac{\Gamma, D \rightarrow C \quad \Gamma \rightarrow D}{\Gamma \rightarrow C}$	$\text{hideL } WL \frac{\Gamma \rightarrow C}{A, \Gamma \rightarrow C}$
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**5: Bellerophon tactic language operators for proof search**

Bellerophon	Operation	Effect
$s \ \& \ t$	sequential composition	run $t$ on the output of $s$ , failing if either fail
$s \   \ t$	alternative choice	run $t$ if applying $s$ failed, failing if both fail
$t^*$	saturating repetition	repeats tactic $t$ until nothing changes any more
$t^*n$	bounded repetition	repeats tactic $t$ exactly $n$ times, failing if any of those repetitions fail
$t^+$	saturating repetition	repeats tactic $t$ at least once until nothing changes any more
$\langle t_1, \dots, t_n \rangle$	branching	runs tactic $t_i$ on branch $i$ , failing if any fail or if branches $\neq n$
$t(j)$	at position	applies tactic $t$ at position $j$ of the sequent
$t(j, e)$	at position	applies tactic $t$ to expression $e$ , which is at position $j$ of the sequent
1	succedent position	position of first succedent formula. Similar 2, 3, ..., 'Rlast
-1	antecedent position	position of first antecedent formula. Similar -2, -3, ..., 'Llast
-4.0.1	subposition	second child of first child of fourth antecedent formula. Similar 4.0.1
'R	search succedent	first applicable succedent position (where formula $e$ is, if specified)
'L	search antecedent	first applicable antecedent position (where formula $e$ is, if specified)
'_	search sequent	first applicable antecedent/succedent position (where $e$ is, if specified)